DISCRETE STRUCTURE LECTURE NOTES-1

BSc.(H) Computer Science: II Semester

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EULER & HAMILTON PATHS

Euler Circuit and Euler Path

- An ***Euler circuit*** in a graph *G* is a simple circuit containing every edge of *G*.
- An ***Euler path*** in *G* is a simple path containing every edge of *G*.

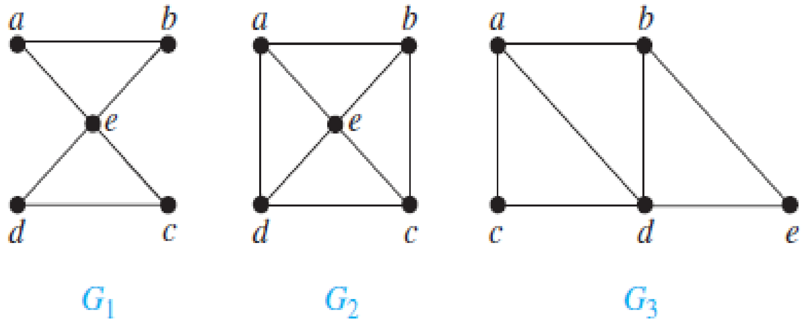

FIGURE 1 **The Undirected Graphs G_1, G_2, and G_3.**

In Figure 1, the graph G1 has an Euler circuit such as a, e, c, d, e, b, a.

Euler Circuit and Euler Path

In Figure 1, the graph G2 has neither Euler circuit nor Euler path. The graph G3 has no Euler circuit but has an Euler path namely, a, c, d, e, b, d, a, b.

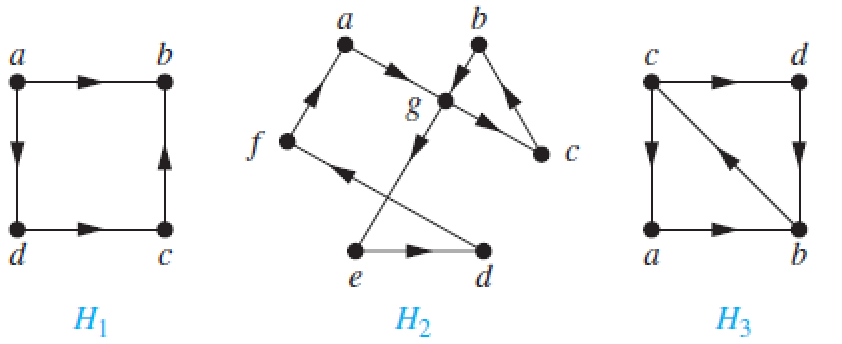

FIGURE 2 **The Directed Graphs H_1, H_2, and H_3.**

In Figure 2, the graph H1 has neither Euler circuit nor Euler path. The graph H2 has Euler circuit such as $a, g, c, b, g, e, d, f, a$. The graph H3 has no Euler circuit but has an Euler path, namely c, a, b, c, d, b.

Euler Path Theorems

- **Theorem 1:** A connected multigraph has an Euler circuit if and only if each vertex has even degree.

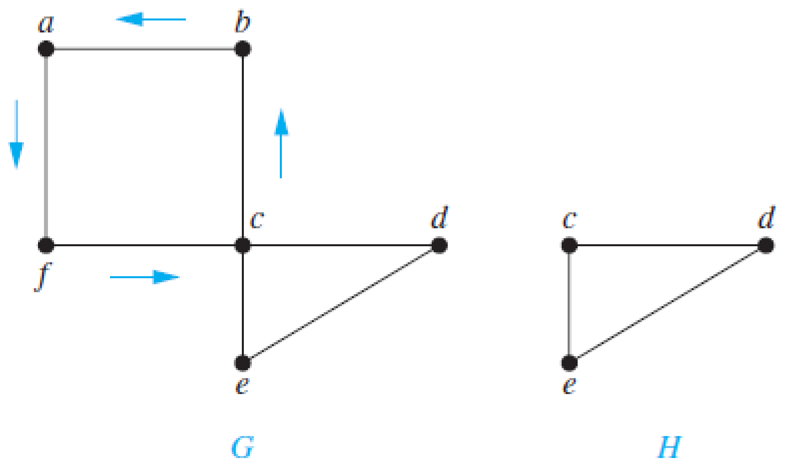

FIGURE 3 **Constructing an Euler Circuit in *G*.**

Euler Path Theorems

- **Theorem 2**: A connected multigraph has an Euler path (but not an Euler circuit) if and only if it has exactly 2 vertices of odd degree.

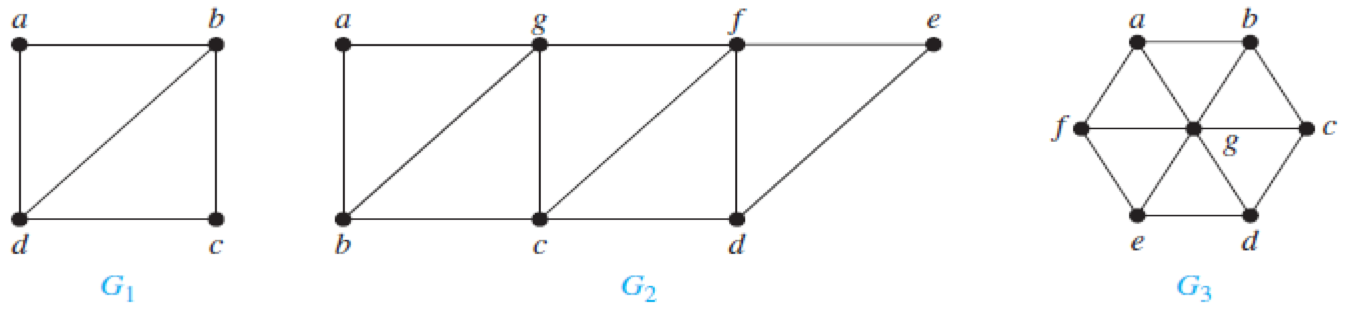

FIGURE 4 **Three Undirected Graphs.**

In Figure 4, graphs G1 and G2 contain exactly two vertices of odd degree namely, b and d. Hence, they must have b and d as its endpoints. Euler paths for G1 and G2 are d, a, b, c, d, b and $b, a, g, f, e, d, c, g, b, c, f, d$ respectively.

Hamilton Circuit and Hamilton Path

- A ***Hamilton circuit*** is a circuit that traverses each vertex in *G* exactly once.
- A ***Hamilton path*** is a path that traverses each vertex in *G* exactly once.

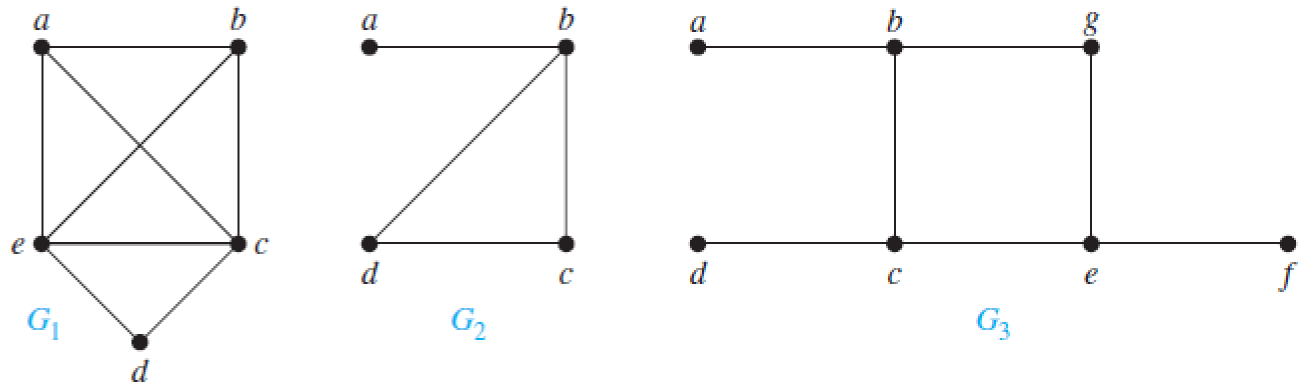

FIGURE 5 **Three Simple Graphs.**

Hamilton Circuit and Hamilton Path

- A ***Hamilton circuit*** is a circuit that traverses each vertex in *G* exactly once.
- A ***Hamilton path*** is a path that traverses each vertex in *G* exactly once.

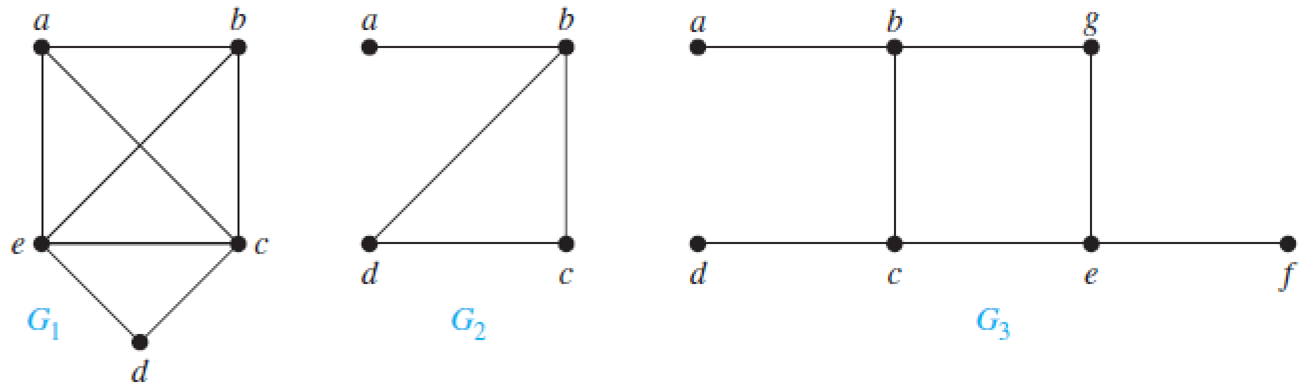

FIGURE 5 Three Simple Graphs.

Hamilton Circuit and Hamilton Path

In Figure 5,

- The graph G1 has a Hamilton circuit a, b, c, d, e, a.
- The graph G2 has no Hamilton circuit but has Hamilton path, namely, a, b, c, d.
- The graph G3 has neither a Hamilton circuit nor a Hamilton path because any containing all vertices must contain one of the edges $\{a, b\}, \{e, f\}$ and $\{c, d\}$.

Hamilton Path Theorems

- **Theorem 1:** If G is a simple path with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then G has Hamilton circuit.

- **Theorem 2:** If G is a simple graph with n vertices with $n \geq 3$ such that $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has Hamilton circuit.

Assignment 1:

Show that K_n has a Hamilton circuit whenever $n \geq 3$.